

# Hybrid Post-Quantum TLS formal specification in Maude-NPA - toward its security analysis

Duong Dinh Tran<sup>1</sup>, Canh Minh Do<sup>1</sup>, Santiago Escobar<sup>2</sup> and Kazuhiro Ogata<sup>1</sup>

<sup>1</sup>*Japan Advanced Institute of Science and Technology (JAIST), Ishikawa, Japan*

<sup>2</sup>*VRAIN, Universitat Politècnica de València, Valencia, Spain*

International Workshop on Formal Analysis and Verification  
of Post-Quantum Cryptographic Protocols 2022

October 24, 2022

# Overview

- Introduction
- Hybrid Post-Quantum TLS
- Maude-NPA formal specification by strands
- Hybrid PQ TLS formal specification
- Analysis experiments
- Summary

# Introduction

- As a response to the quantum attack threat, AWS has proposed the Hybrid Post-Quantum (PQ) Transport Layer Security (TLS) Protocol\*, a quantum-resistant version of TLS 1.2.
- The “hybrid” terminology refers to the hybrid key exchange mode used in the protocol:
  - a conventional key exchange algorithm, fixed as Elliptic Curve Diffie-Hellman (ECDH), and
  - a post-quantum key encapsulation mechanism (KEM), e.g., Kyber.



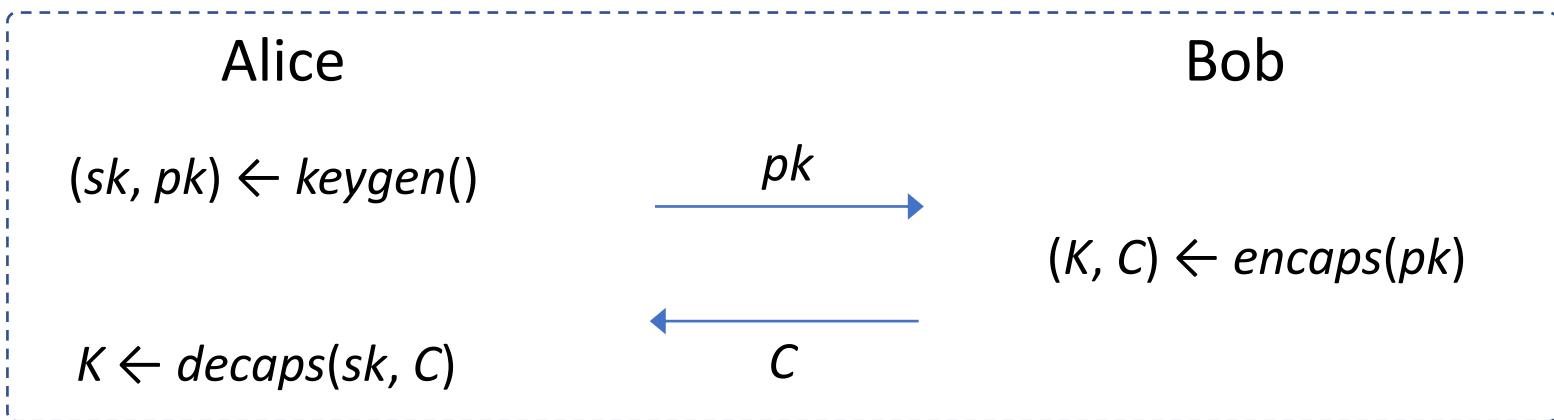
→ Hybrid PQ TLS: Formal specification and analysis with Maude-NPA

\*<https://datatracker.ietf.org/doc/html/draft-campagna-tls-bike-sike-hybrid>

# Key Encapsulation Mechanism (KEM)

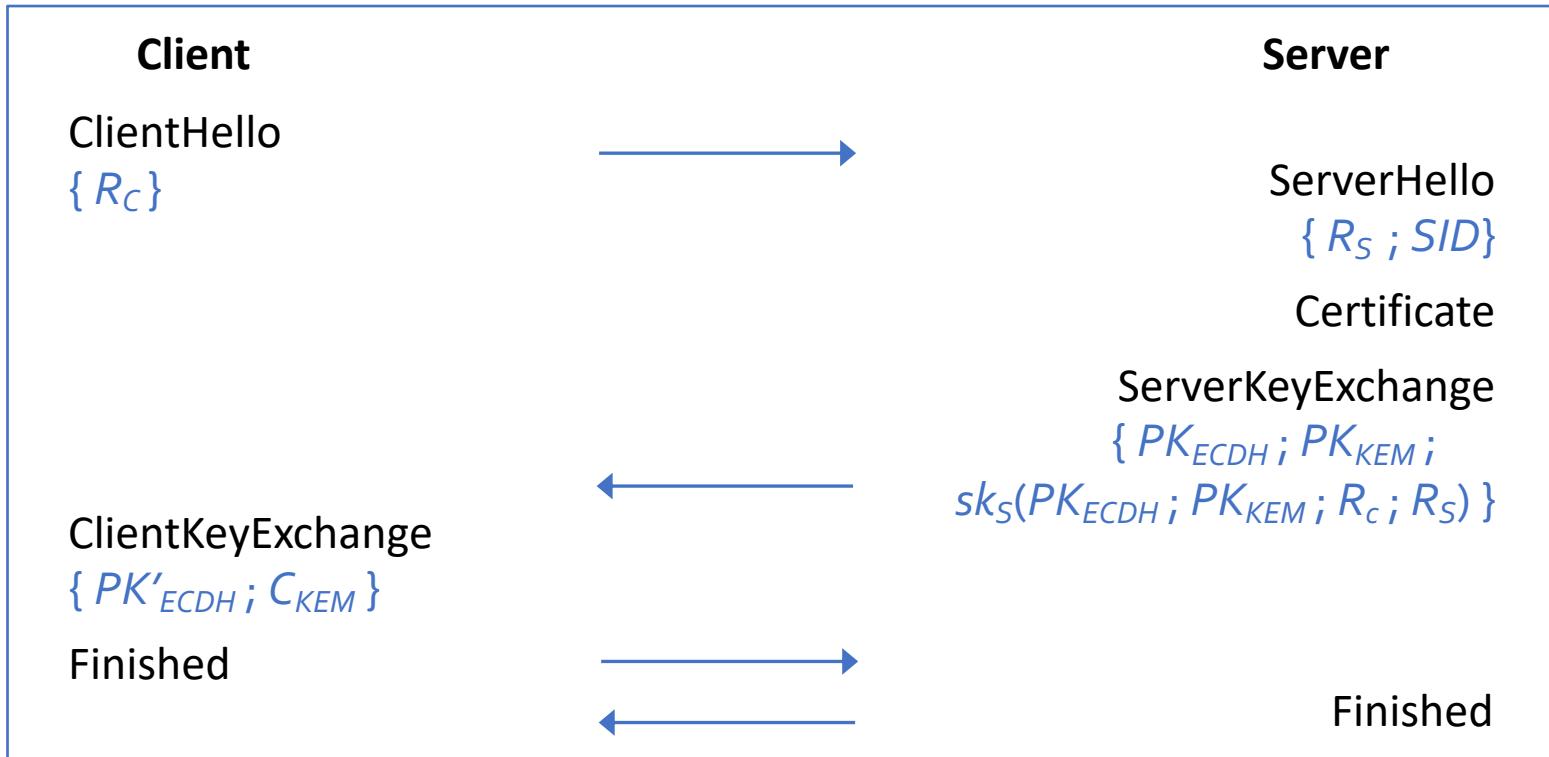
A KEM is a tuple of algorithms ( $\text{keygen}$ ,  $\text{encaps}$ ,  $\text{decaps}$ ):

1.  $(sk, pk) \leftarrow \text{keygen}()$ : outputs a public key  $pk$  and a secret key  $sk$ .
2.  $(K, C) \leftarrow \text{encaps}(pk)$ : takes the public key  $pk$ , and outputs a ciphertext  $C$  and a shared secret key  $K$ .
3.  $K \leftarrow \text{decaps}(sk, C)$ : takes the secret key  $sk$ , a ciphertext  $C$  and outputs the shared secret key  $K$ .



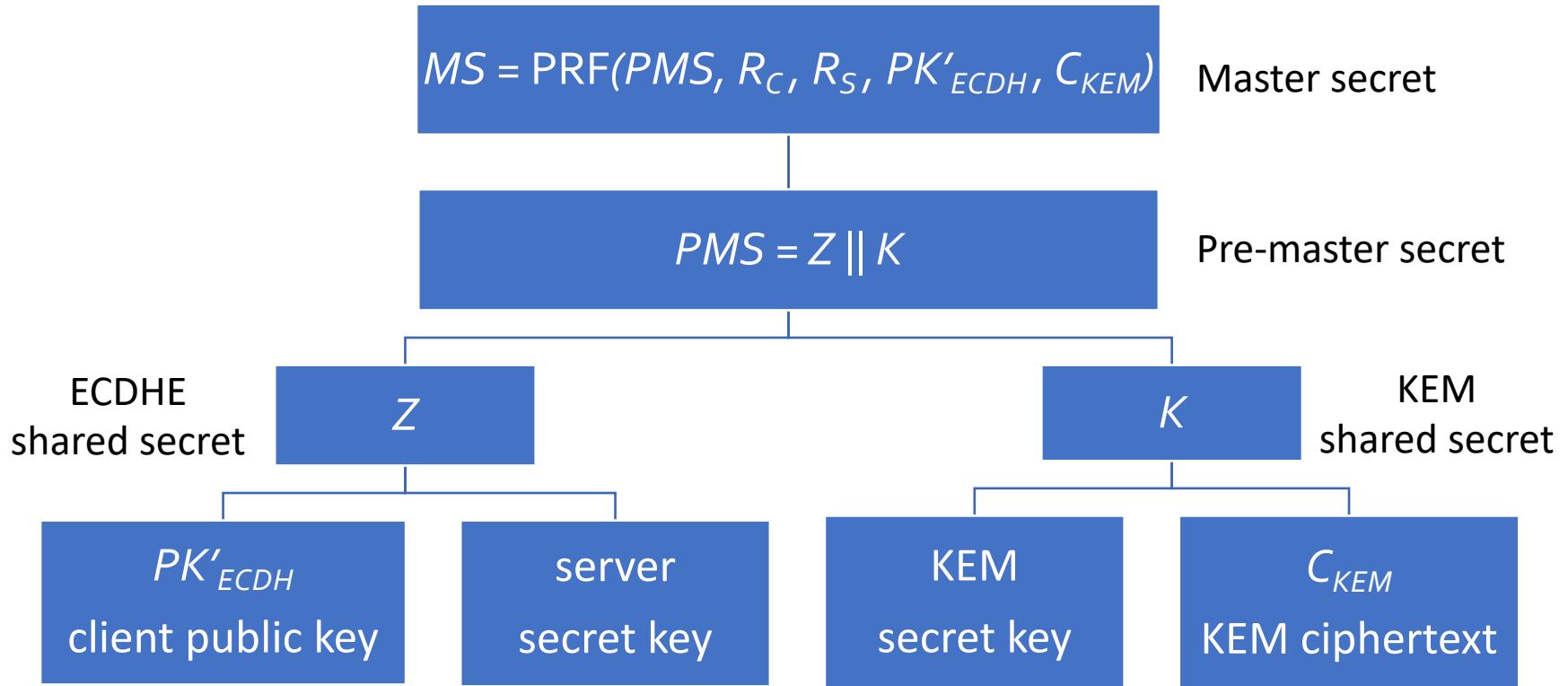
# Hybrid PQ TLS protocol

We consider a simplified version of the Hybrid PQ TLS as follows:



- $sk_S(PK_{ECDH} ; PK_{KEM} ; R_c ; R_s)$ : encrypt the plaintext by the private key of  $S$ .
- Finished message: hash of all messages before encrypted by the handshake key negotiated, whose calculation is explained in the next slide.

# Key calculation



- PRF: Pseudorandom function
- We use the master secret as the symmetric key for encryption of the Finished messages

# Maude-NPA formal specification by strands

A strand is in the following form:

$$:: r_1, \dots, r_k :: [+(m_1), -(m_2), \dots, -(m_i) \mid +(m_{i+1}), \dots ]$$

- $r_1, \dots, r_k$  denote unique freshes generated in the strand.
- $+(m)$  and  $-(m)$  denote sending and receiving the message  $m$ , respectively.
- messages appearing before “ $\mid$ ” were sent/received in the past.
- messages appearing after “ $\mid$ ” will be sent/received in the future.

Let's consider the Needham-Schroeder Public Key (NSPK) protocol:

- |  |   |   |
|--|---|---|
| i) $A \rightarrow B : pk(B, A ; N_A)$    | ; | denotes the concatenation                       |
| ii) $B \rightarrow A : pk(A, N_A ; N_B)$ |   | $N_A$ : unique nonce generated by $A$           |
| iii) $A \rightarrow B : pk(B, N_B)$      |   | $pk(A, m)$ : $m$ encrypted by $A$ 's public key |

# Maude-NPA formal specification by strands

--- *principal names and nonces*

**sorts** Name Nonce .

**subsort** Name Nonce < Msg .

--- *a nonce is in form of n(A,r), where r (of sort Nonce) guarantees its uniqueness*

**op** n : Name Fresh -> Nonce [frozen] .

--- *public & private encryption*

**op** pk : Name Msg -> Msg [frozen] .

**op** sk : Name Msg -> Msg [frozen] .

Strand specifying the protocol execution from the A side:

```
:: r :: [ nil | +(pk(B, A ; n(A,r))), -(pk(A, n(A,r) ; N)), +(pk(B, N)), nil ]
```

Strand specifying the protocol execution from the B side:

```
:: r :: [ nil | -(pk(B, A ; N)), +(pk(A, N ; n(B,r))), -(pk(B, n(B,r))), nil ]
```

# Hybrid PQ TLS specification: KEMs

--- *public keys, secret keys, encapsulations, and shared keys*

**sorts**      PqPk PqSk Cipher PqKey

**subsort**    PqPk PqSk Cipher PqKey < Msg .

**op** pqSk    : Name Fresh -> PqSk [frozen] .

**op** pqPk    : PqSk                -> PqPk [frozen] .

**op** \$pqKey : PqSk PqSk    -> PqKey [frozen] .

**op** encapCipher : PqPk PqSk -> Cipher [frozen] .      --- *encaps*

**op** encapKey    : PqPk PqSk -> PqKey [frozen] .      --- *encaps*

**op** decap : Cipher PqSk -> PqKey [frozen] .            --- *decaps*

--- *algebraic properties of KEMs*

**eq** pqKey(pqPk(S:PqSk), S2:PqSk) = \$pqKey(S:PqSk, S2:PqSk) [variant] .

**eq** decap(encapCipher(pqPk(S:PqSk), S2:PqSk), S:PqSk)

= \$pqKey(S:PqSk, S2:PqSk) [variant] .

# Hybrid PQ TLS specification: ECDH

--- points (on the curve), scalars, and ECDH shared keys

**sorts** Point Scalar ECKey .

**subsort** Point < ECKey .

**op** p : -> Point .

--- the point generator

**op** sk : Name Fresh -> Scalar [frozen] . --- a unique scalar serving as a secret key

--- gen(p, s) produces a public key pk (to send to the opposite peer)

--- gen(pk, s) produces a shared key (where pk is received from the opposite peer)

**op** gen : Point Scalar -> Point [frozen] .

--- multiplication on scalars.

**op** \_\*\_ : Scalar Scalar -> Scalar [frozen assoc comm] .

--- algebraic properties of ECDH

**eq** gen(gen(P:Point, K1:Scalar), K2:Scalar)

= gen(P:Point, K1:Scalar \* K2:Scalar) [variant] .

# Hybrid PQ TLS specification

--- *Pre-Master Secrets & Master Secrets*

**op** pms : ECKey PqKey -> PreMasterSecret [frozen] .

**op** ms : PreMasterSecret Rand Rand ECKey Cipher -> MasterSecret [frozen] .

--- *signature, certificates, randoms, and session IDs*

**op** sig : Name Msg -> Msg [frozen] .

**op** cert : Name -> Cert [frozen] .

**op** rd : Name Fresh -> Rand [frozen] .

**op** sess : Name Fresh -> Session [frozen] .

--- *encryption & decryption*

**op** enc : MasterSecret Msg -> Msg [frozen] .

**op** dec : MasterSecret Msg -> Msg [frozen] .

# Protocol execution specification: client side

```
eq STRANDS-PROTOCOL
= :: r1,r2,r3 ::

[ nil |
+(ch ; rd(C,r1)),
-(sh ; N ; SS),
-(sc ; cert(S)),
-(ske ; PK1 ; PK2 ; sig(S, PK1 ; PK2 ; rd(C,r1) ; N)),
+(cke ; gen(p, sk(C,r2)) ; cipher(PK2, pqSk(C,r3))),
+(cf ; enc(ms(pms(gen(PK1, sk(C,r2)), pqKey(PK2, pqSk(C,r3)) ),
rd(C,r1), N, gen(p,sk(C,r2)), cipher(PK2, pqSk(C,r3)) )),
(ch ; rd(C,r1)) ++
(sh ; N ; SS) ++
(sc ; cert(S)) ++
(ske ; PK1 ; PK2 ; sig(S, PK1 ; PK2 ; rd(C,r1) ; N)) ++
(cke ; gen(p,sk(C,r2)) ; cipher(PK2, pqSk(C,r3)) )
),
...
...
```

# Protocol execution specification: server side

```
:: r1,r2,r3,r4 ::  
[ nil |  
-(ch ; N),  
+(sh ; rd(S,r1) ; sess(S,r2)),  
+(sc ; cert(S)),  
+(ske ; gen(p,sk(S,r3)) ; pqPk(pqSk(S,r4)) ;  
    sig(S, gen(p,sk(S,r3)) ; pqPk(pqSk(S,r4)) ; N ; rd(S,r1))),  
-(cke ; PK1 ; CP),  
-(cf ; enc(ms(ms(gen(PK1,sk(S,r3)), decap(CP, pqSk(S,r4))) ),  
    N, rd(S,r1), PK1, CP ),  
  (ch ; N) ++  
  (sh ; rd(S,r1) ; sess(S,r2)) ++  
  (sc ; cert(S)) ++  
  (ske ; gen(p,sk(S,r3)) ; pqPk(pqSk(S,r4)) ;  
    sig(S, gen(p,sk(S,r3)) ; pqPk(pqSk(S,r4)) ; N ; rd(S,r1))) ++  
  (cke ; PK1 ; CP ) )  
, ...
```

# Intruder capabilities

--- *messages concatenation & deconcatenation*

```
:: nil :: [ nil | -(M1 ; M2), +(M1), nil ] &  
:: nil :: [ nil | -(M1 ; M2), +(M2), nil ] &  
:: nil :: [ nil | -(M1), -(M2), +(M1 ; M2), nil ]
```

--- *generate any random number and any point*

```
:: r :: [ nil | +(rd(i,r)), nil ] &  
:: r :: [ nil | +(sk(i,r)), nil ]
```

--- *with KEMs*

```
:: nil :: [ nil | -(PK2), -(SK), +(encapCipher(PK2,SK)), nil ] &  
:: nil :: [ nil | -(PK2), -(SK), +(encapKey(PK2,SK)), nil ] &  
:: nil :: [ nil | -(CP), -(SK), +(decap(CP,SK)), nil ]
```

--- *with ECDH, we suppose that the intruder can break*

--- *the key exchange security by utilizing the power of quantum computers*

```
:: nil :: [ nil | -(gen(p,K1)), -(gen(p,K2)), +(gen(p,K1 * K2)), nil ]
```

# Analysis: learning ECDH shared key

```
eq ATTACK-STATE(0)
= :: r1,r2,r3 ::

[ nil,
+(ch ; rd(c,r1)),
-(sh ; rd(s,r1') ; sess(s,r'))),
-(sc ; cert(s)),
-(ske ; gen(p, sk(s,r2')) ; pqPk(pqSk(s,r3')) ; . . . ),
+(cke ; gen(p, sk(c,r2)) ; cipher(pqPk(pqSk(s,r3')), pqSk(c,r3)) ),
+(cf ; . . .),
-(sf ; . . .) | nil ]
|| gen(p, sk(s,r2') * sk(c,r2)) inl, empty           --- ECDH shared secret key
|| nil
|| nil
|| nil
[nonexec].
```

# Analysis: secrecy property

```
eq ATTACK-STATE(1)
= :: r1,r2,r3 ::

[ nil,
+(ch ; rd(c,r1)),
-(sh ; rd(s,r1') ; sess(s,r')),  

-(sc ; cert(s)),
-(ske ; gen(p,sk(s,r2')) ; pqPk(pqSk(s,r3')) ; . . . ),
+(cke ; gen(p,sk(c,r2)) ; cipher(pqPk(pqSk(s,r3')), pqSk(c,r3)) ),
+(cf ; . . .),
-(sf ; . . .) | nil ]
|| $pqKey(pqSk(s,r3'), pqSk(c,r3)) inl, empty    --- PQ KEM shared secret key
|| nil
|| nil
|| nil
[nonexec].
```

# Analysis: experiment results

ATTACK-STATE	Result/Bound	Maude-NPA (h:m:s)	Par-Maude-NPA (h:m:s)	Percentage of improvement
0	X/12	18:23:01	05:35:06	69.6 %
1	✓/12	06:24:29	02:00:06	68.8 %

Thank you for your attention!